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| 14. ABSTRACT  |                   |                                |   |  |   |
| 15. SUBJECT TERMS   |                   |                                |   |  |   |
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**RPPR Final Report**  
as of 09-Nov-2017

Agency Code:

Proposal Number: 69646MACF

**Agreement Number: W911NF-16-1-0394**

**INVESTIGATOR(S):**

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**Report Date:** 31-Oct-2017

Date Received: 08-Nov-2017

**Final Report** for Period Beginning 01-Aug-2016 and Ending 31-Jul-2017

**Title:** Future Directions in Fractional Calculus Research and Applications

**Begin Performance Period:** 01-Aug-2016

**End Performance Period:** 31-Jul-2017

**Report Term:** 0-Other

Submitted By: Ph.D. Mark Meerschaert

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**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

**STEM Degrees:**

**STEM Participants:**

**Major Goals:** A week-long workshop on Future Directions in Fractional Calculus Research and Applications at Michigan State University, was held on 17 to 21 October 2016.

**Accomplishments:** A week-long workshop on Future Directions in Fractional Calculus Research and Applications at Michigan State University, was held on 17 to 21 October 2016. The event was a huge success, including distinguished speakers at the very top of the field from the USA and across the world. We also had a poster session for young investigators. A special issue was published including many papers from the conference.

**Training Opportunities:** At least ten postdocs graduate students were able to attend all or part of the conference, interact with the invited speakers, and participate in a poster session. We also made the slides for all the talks available on a conference web site.

**Results Dissemination:** A conference web site <https://stt.msu.edu/FCworkshop/> was set up and is being maintained by the host institution, Michigan State University including the entire program with links to the abstract and slides for each talk, see Upload section of this report..

A special issue of the journal Chaos, Solitons & Fractals was published:

Chaos, Solitons & Fractals

Volume 102, Pages 1-486 (September 2017)

Future Directions in Fractional Calculus Research and Applications

Edited by MARK M. MEERSCHAERT, BRUCE J. WEST and YONG ZHOU

The special issue included 53 articles, including most of the papers presented at the conference. Some articles were also accepted from leading figures in the field who were not able to attend the conference.

**Honors and Awards:** PI Mark M. Meerschaert was named University Distinguished Professor on 21 June 2017, the highest honor at Michigan State University.

Young researcher James F. Kelly was awarded the Bruce J. West award for best paper at the workshop funded by this grant. The award was determined by a vote of the invited speakers.

**RPPR Final Report**  
as of 09-Nov-2017

**Protocol Activity Status:**

**Technology Transfer:** Nothing to Report



## Schedule

### 17 October 2016 (Monday)

Morning:

9:00 Bruce West: Complexity Science and Fractional Calculus [[Abstract](#) - Presentation ([PPTX](#) / [PDF](#))]

10:00 Alla Sikorskii: Applications of Fractional Calculus to Stochastic Models for Finance [[Abstract](#) - [Presentation](#)]

11:00 Ervin Lenzi: Poisson-Nernst-Planck Diffusional Model and Fractional Time Derivatives: Applications to Electrical Response [[Abstract](#)]

*12:00 lunch break*

Afternoon:

2:00 ZhenQing Chen: Anomalous Diffusions and Fractional Order Differential Equations [[Abstract](#) - [Presentation](#)]

3:00 Richard Magin: A Fractional Derivative Model of Anomalous Diffusion in White and Gray Matter [[Abstract](#) - [Presentation](#)]

4:00 Clara Ionescu: How the Fractional Order Impedance Models Influenced Lung Function Device Trends [[Abstract](#) - [Presentation](#)]

### 18 October 2016 (Tuesday)

Morning:

9:00 Zhi Zhou: Numerical Analysis for Time-Fractional Evolution Equations [[Abstract](#) - [Presentation](#)]

10:00 Mohsen Zayernouri: Data-Driven FPDE Modeling and Simulation [[Abstract](#)]

11:00 Mark Ainsworth: Analysis and Approximation of a Fractional Cahn-Hilliard Equation [[Abstract](#)]

*12:00 lunch break*

Afternoon:

2:00 Nick Laskin: Fractional Quantum Mechanics [[Abstract](#)]

3:00 Changpin Li: The Finite Difference Method for Caputo-type Parabolic Equation with Fractional Laplacian [[Abstract](#)]

4:00 Hong Wang: Fast Numerical Methods and Mathematical Analysis of Fractional Partial Differential Equations [[Abstract](#) - [Presentation](#)]

5:00 Poster Session

[A Petrov-Galerkin Spectral Element Method for Fractional Elliptic Problems](#)  
[Density Bounds for some Degenerate Stable Driven SDEs.](#)

[Boundary Conditions for FPDE on a Finite Interval](#)

19 October 2016 (Wednesday)

Morning:

9:00 David Benson: The Richness of Fractional Integro-Differential Operators Defined by Convolution with the Levy Measure [[Abstract](#) - [Presentation](#)]

10:00 Diogo Bolster: Fractional Dispersion and Mixing Driven Reactions [[Abstract](#) - [Presentation](#)]

11:00 Rina Schumer: Anomalous Transport, Rough Landscapes, And Preservation of Stratigraphy [[Abstract](#)]

*12:00 lunch break*

Afternoon:

2:00 Weihua Deng: Mean Exit Time and Escape Probability for the Anomalous Processes with the Tempered Power-law Waiting Times [[Abstract](#)]

3:00 Vaughan Voller: Promoting the Use of Fractional Calculus in the Modeling Engineering Systems [[Abstract](#)]

4:00 Yong Zhang: Applications, Challenges, and Suggestions of Fractional-Derivative Models in Simulating Hydrologic Processes [[Abstract](#)]

20 October 2016 (Thursday)

Morning:

9:00 Qiang Du: Nonlocal Diffusion Models as Bridges Between Local and Fractional Diffusion Models [[Abstract](#)]

10:00 Erkan Nane: Space-Time Fractional Stochastic Partial Differential Equations [[Abstract](#) - [Presentation](#)]

11:00 Lorenzo Toniazzi: On a Probabilistic Generalization of Fractional Derivatives [[Abstract](#) - [Presentation](#)]

*12:00 lunch break*

Afternoon:

2:00 Sergei Fedotov: Nonlinear Fractional PDE's and Their Applications in Biology [[Abstract](#) - [Presentation](#)]

3:00 Renat Sibatov: Fractional Derivatives On Cosmic Scales [[Abstract](#) - [Presentation](#) ([PPT](#) / [PDF](#))]

4:00 HongGuang Sun: Recent Advantages and Open Issues in Anomalous Diffusion Models and Its Numerical Methods [[Abstract](#)]

21 October 2016 (Friday)

Morning:

9:00 George Karniadakis: Spectral and High-Order Methods for Solving Fractional PDEs, Smooth and Non-Smooth Solutions

10:00 Gary Bohannon: Preparing the Next Generation of Researchers [[Abstract](#)]

11:00 YangQuan Chen: Regional Sensing and Actuation of Fractional Order Distributed Parameter Systems [[Abstract](#) - [Presentation](#)]

12:00 *lunch break*

Afternoon:

2:00 J. Tenreiro Machado: A Stranger in a Strange Fractional Land [[Abstract](#) - [Presentation](#)]

3:00 Enrico Scalas: Pseudo-Differential Relaxation Equations and Semi-Markov Processes [[Abstract](#) - [Presentation](#)]

4:00 Mark Meerschaert: Space-Time Duality and Medical Ultrasound [[Abstract](#) - [Presentation](#)]